Weaker temperature gradient in a warmer climate

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PRINCETON UNIVERSITY

The weak temperature gradient approximation (Charney 1963, *JAS*)

For large-scale ($L \approx 1000km$) motion in tropical free-troposphere:

$$\frac{\partial \vec{u}}{\partial t} + \vec{v} \cdot \nabla \vec{u} + f\hat{z} \times \vec{u} = -\frac{1}{\rho} \nabla_z p$$

$$\downarrow$$

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$$\left.\begin{aligned} \vec{v} \cdot \nabla \vec{u} &\approx -\frac{1}{\rho} \nabla_z p \\ \mathrm{d}p &= -\frac{p}{R_d T_v} g \mathrm{d}z \end{aligned}\right\} \quad \frac{\delta T_v}{T_v} \sim \frac{\delta p}{p} \sim \frac{U^2}{gH} \approx 10^{-3}$$

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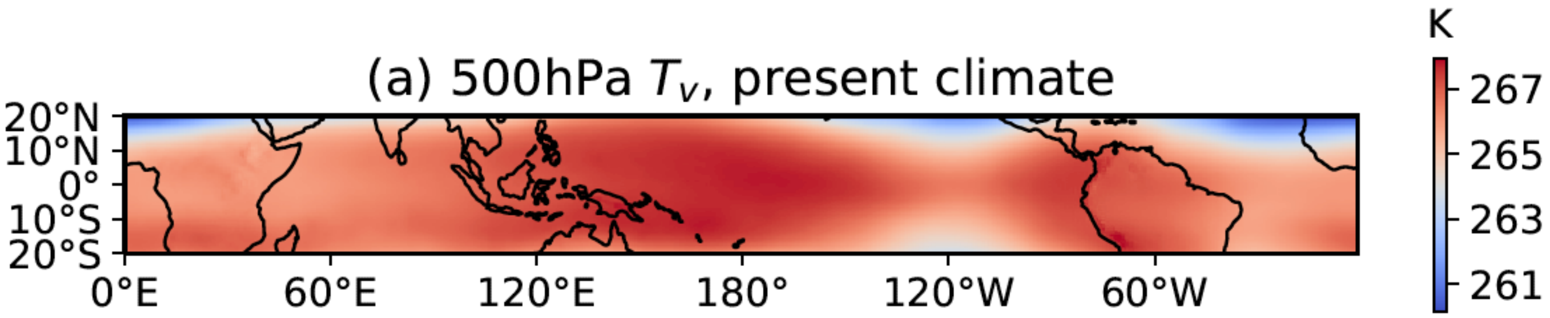

Virtual temperature
$T_v = T(1 + 0.61q)$

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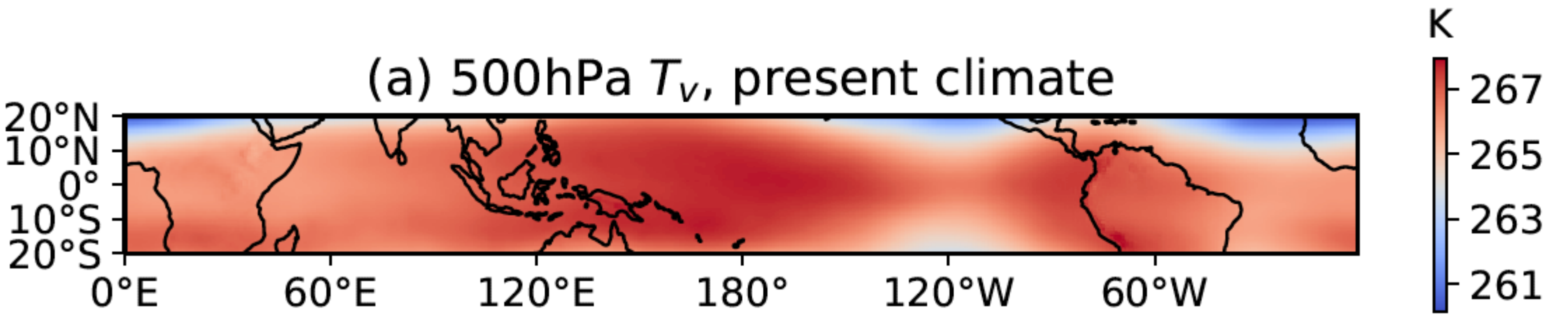
Question: How will tropical free troposphere temperature gradient respond to global warming?



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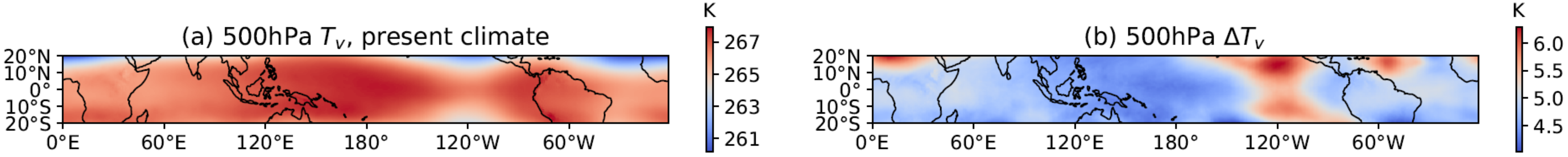
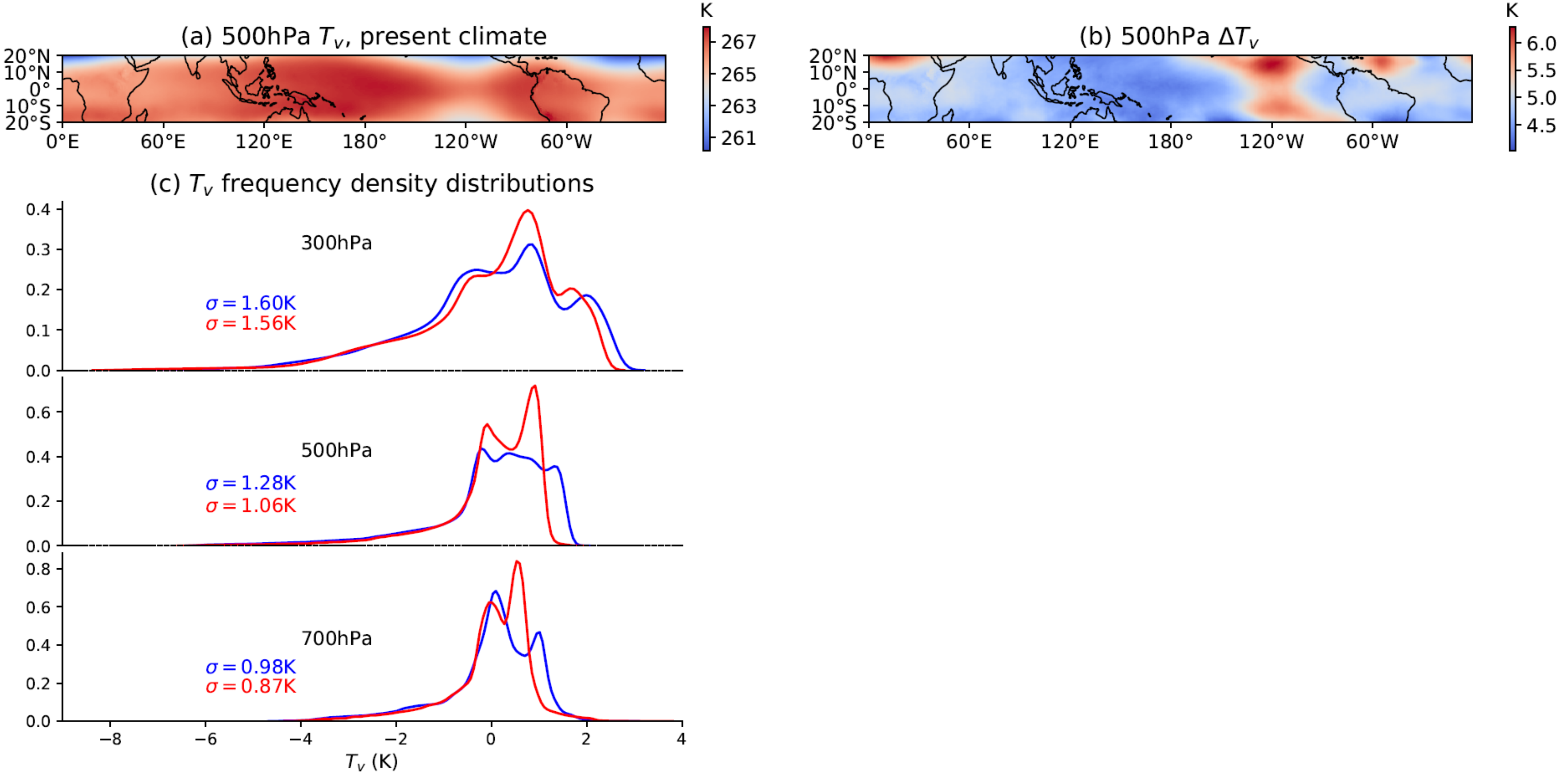
GCM shows weaker temperature gradient in a warmer climate



CM2.5-FLOR, +1% CO2/yr
Present climate: yr 1~10 January average
Warmer climate: yr 131~140 January average

Δ: warmer climate – present climate

GCM shows weaker temperature gradient in a warmer climate



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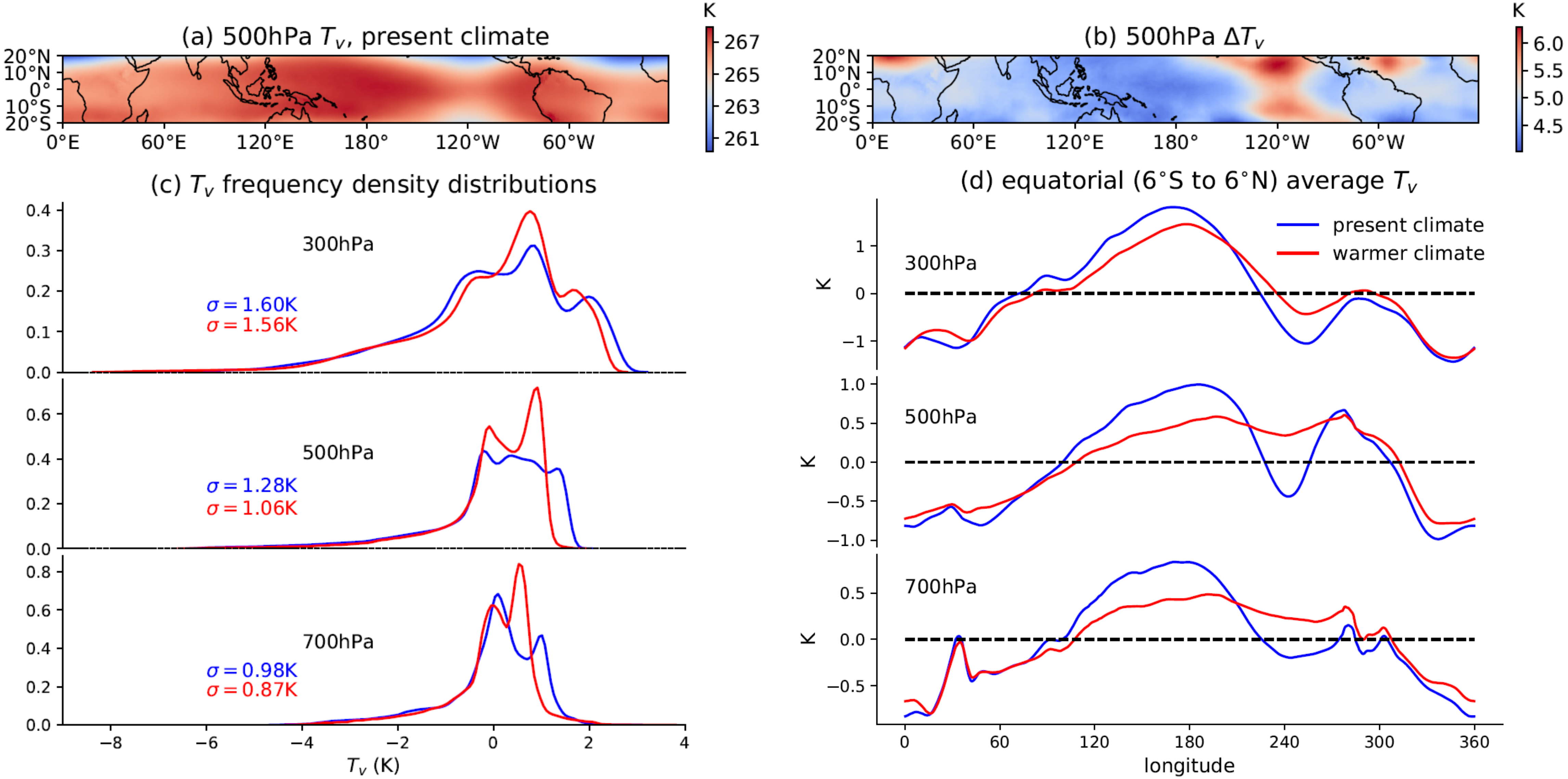

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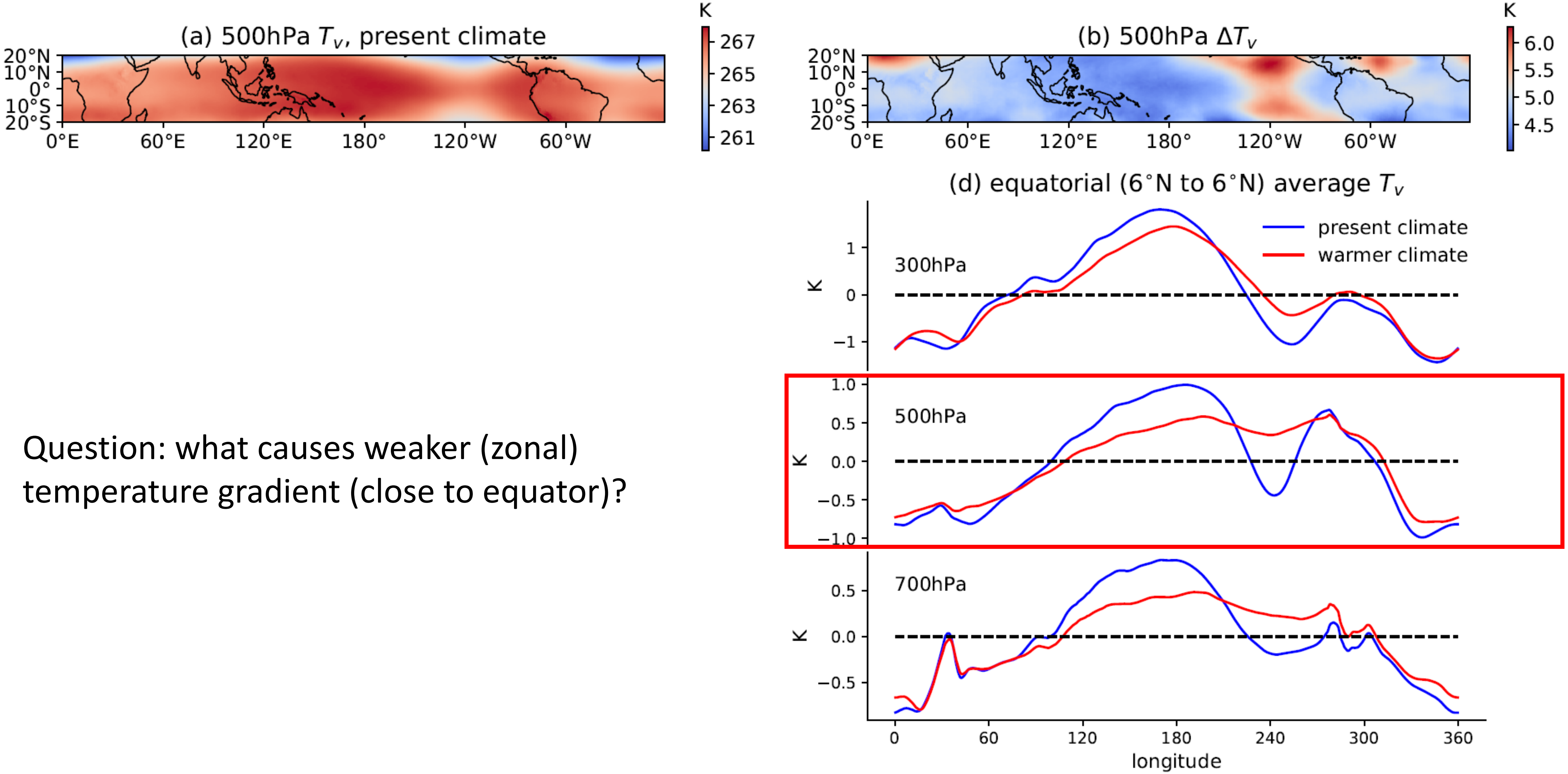
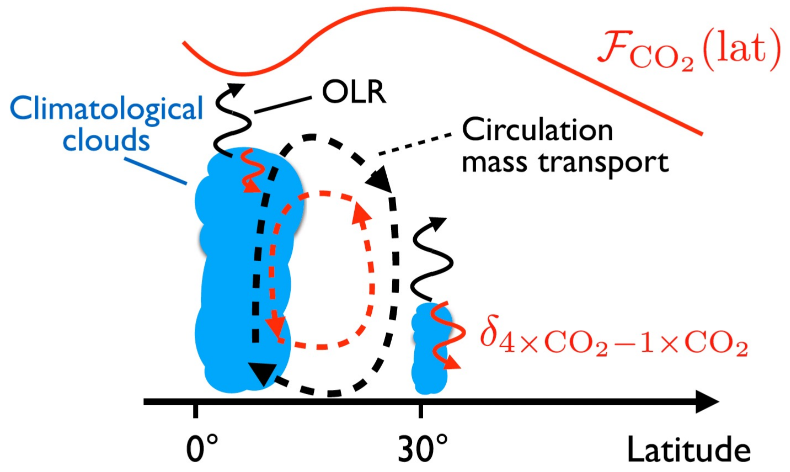


Question: what causes weaker (zonal) temperature gradient (close to equator)?

Hypothesis 1: masked CO2 forcing



Merlis 2015, *PNAS*

Larger CO2 forcing in subsiding regions causes greater free troposphere warming?

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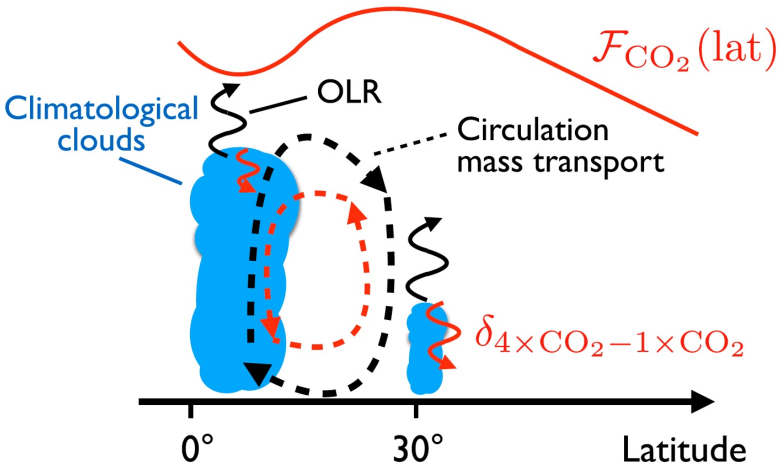

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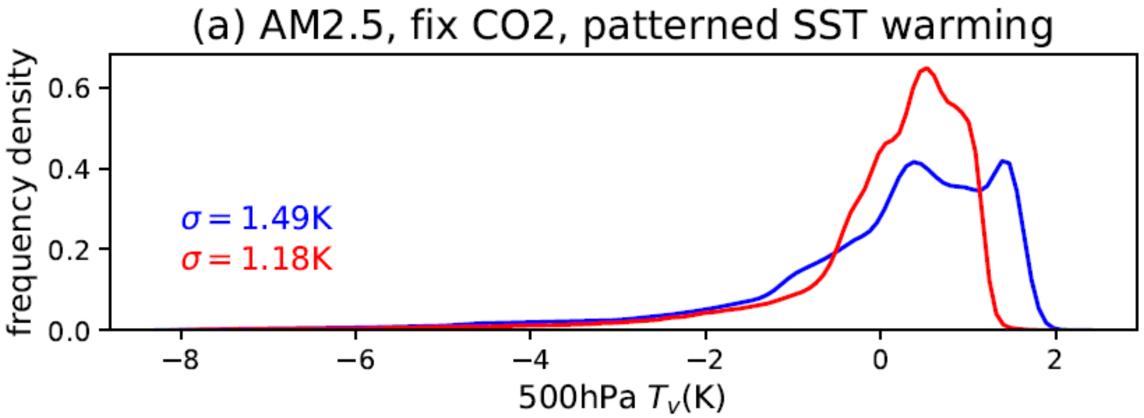
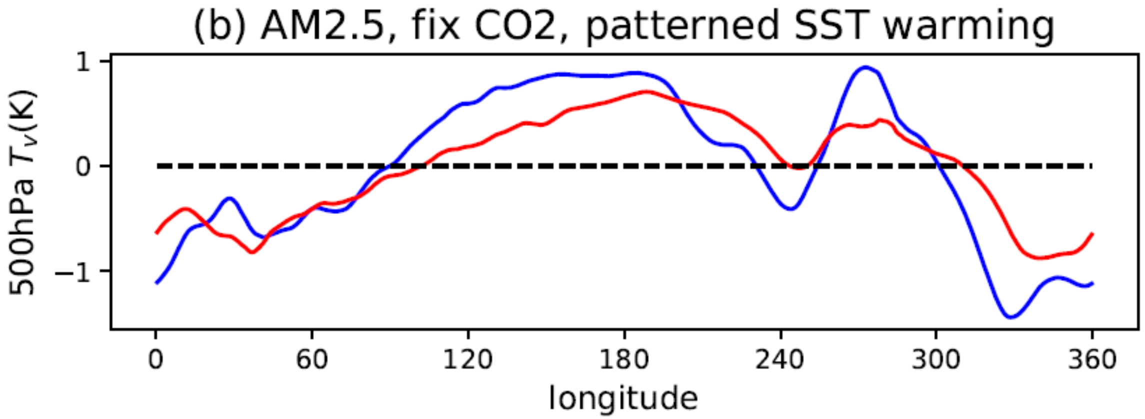
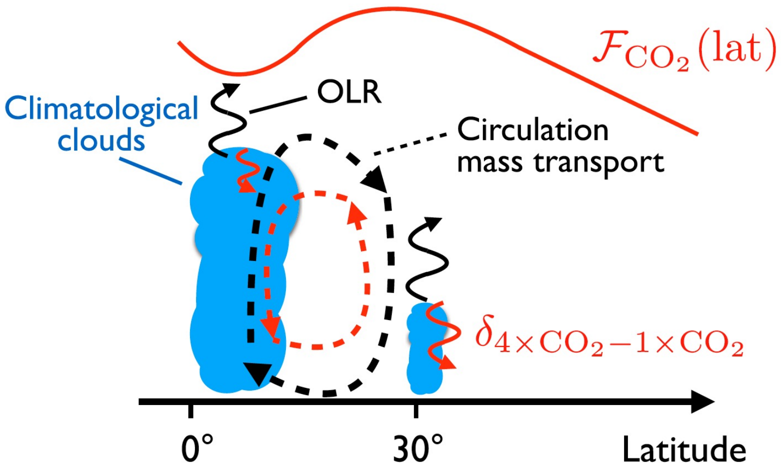




Without CO2 forcing, temperature gradient still becomes weaker

Hypothesis 1: masked CO2 forcing ❌



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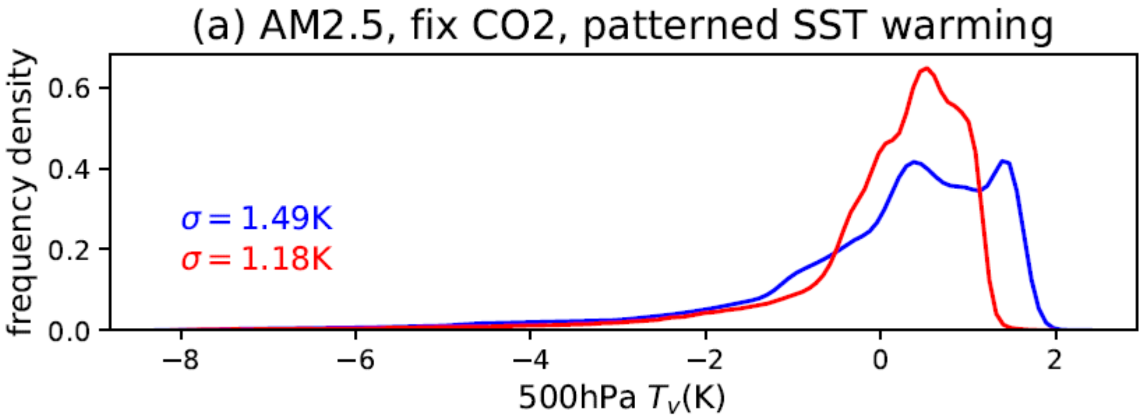

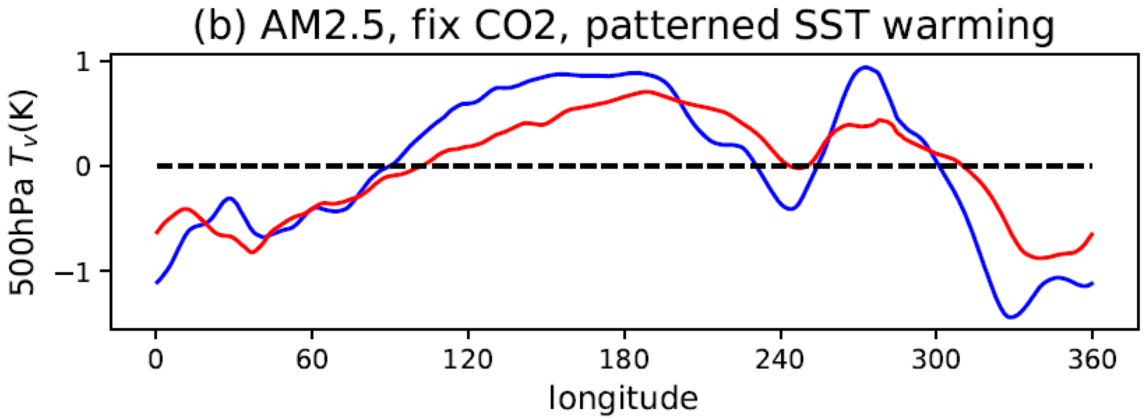

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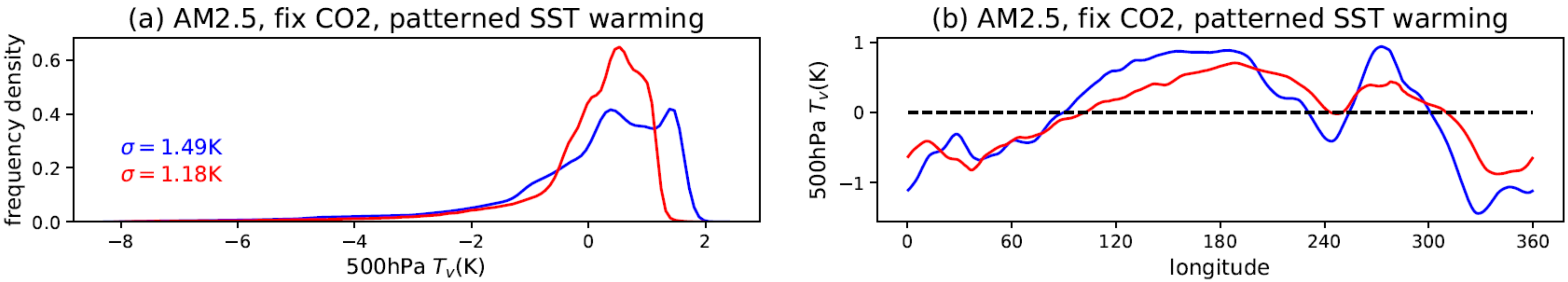
Hypothesis 2: patterned SST warming

Larger surface warming in subsiding regions → larger free-troposphere warming?



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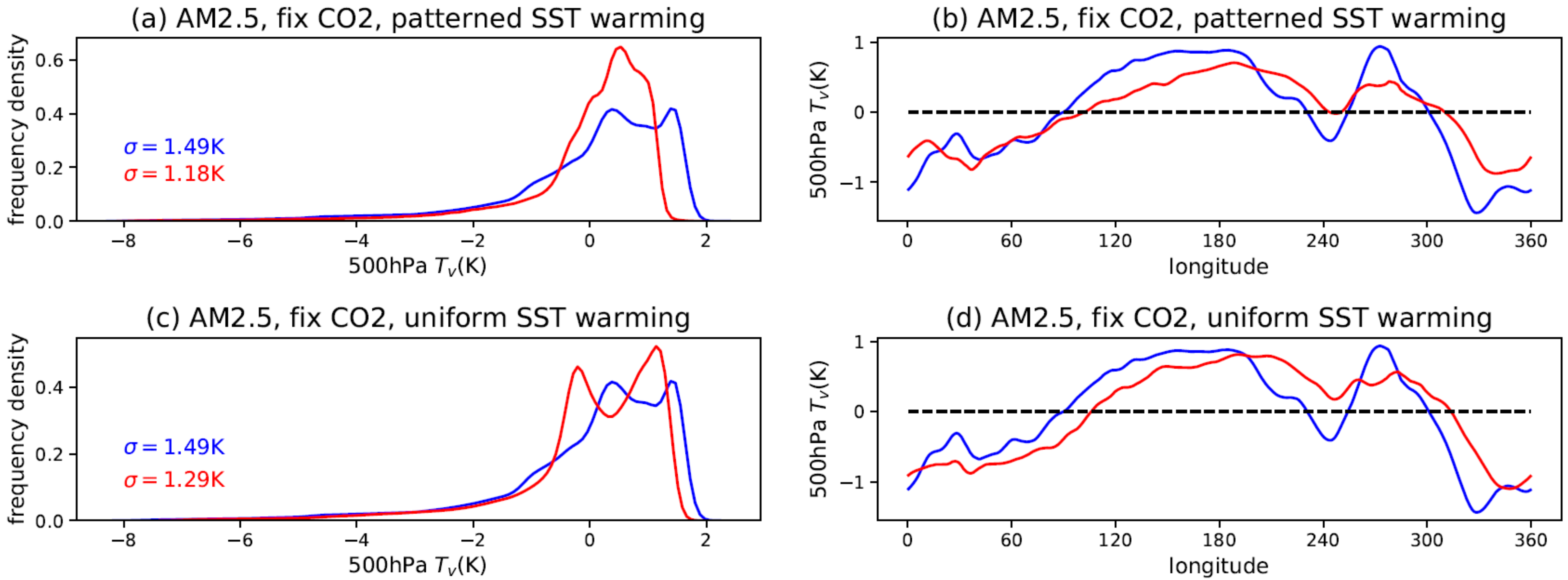

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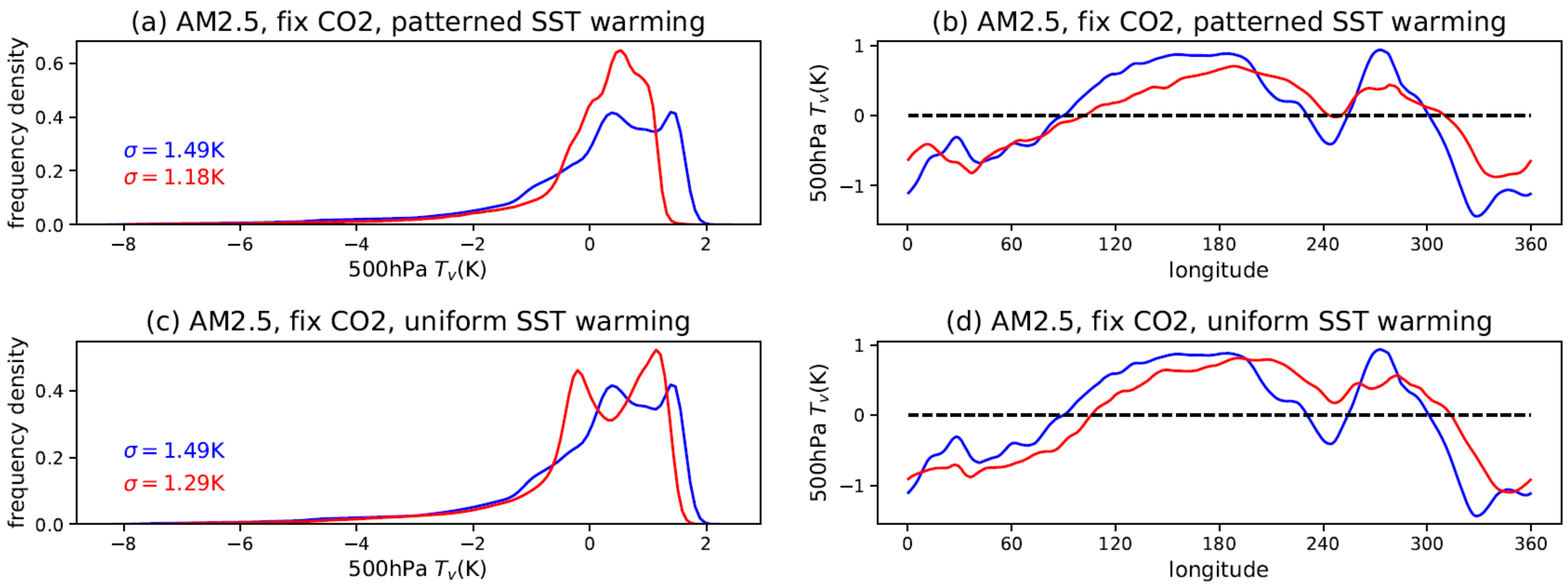

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Hypothesis 3: circulation weakening ✅

$\frac{\partial \Phi}{\partial x} \approx (-\vec{v} \cdot \nabla u)$ close to equator (Bao et al., 2022, *JCLI*)

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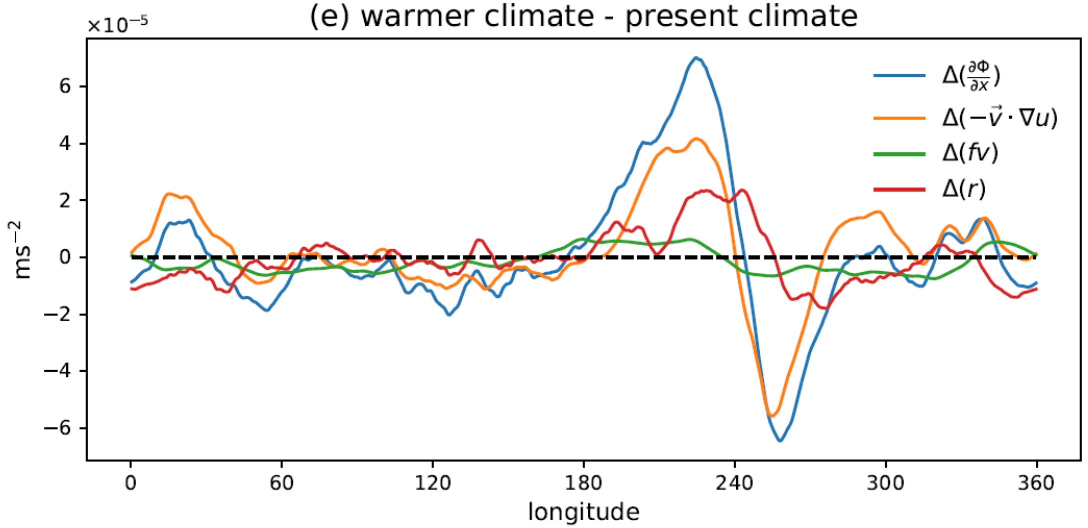

$\Delta(\frac{\partial \Phi}{\partial x}) \approx \Delta(-\vec{v} \cdot \nabla u)$ under global warming

Weaker tropical circulation (Held & Soden, 2006, *JCLI*)

→ Weaker momentum advection

→ Weaker pressure (temperature) gradient

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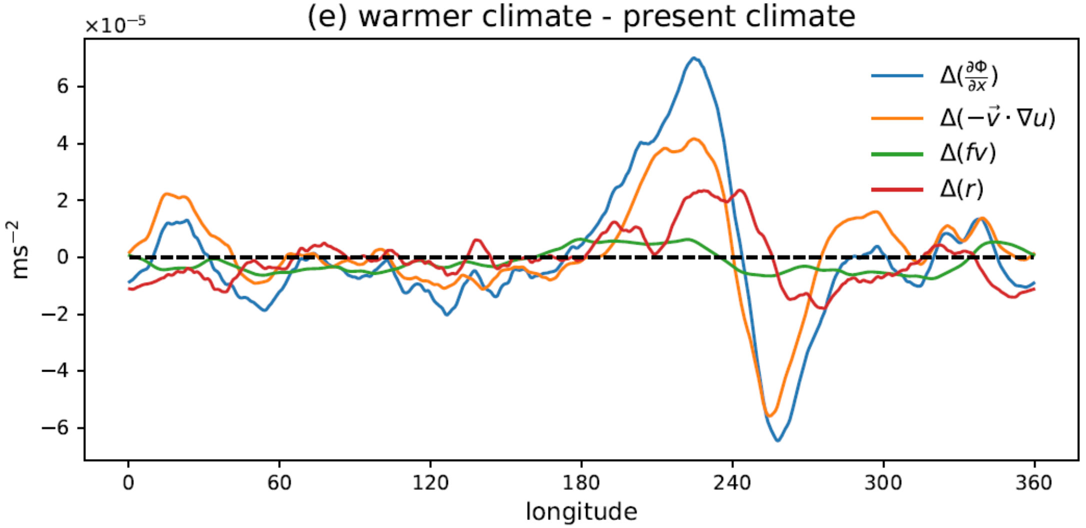

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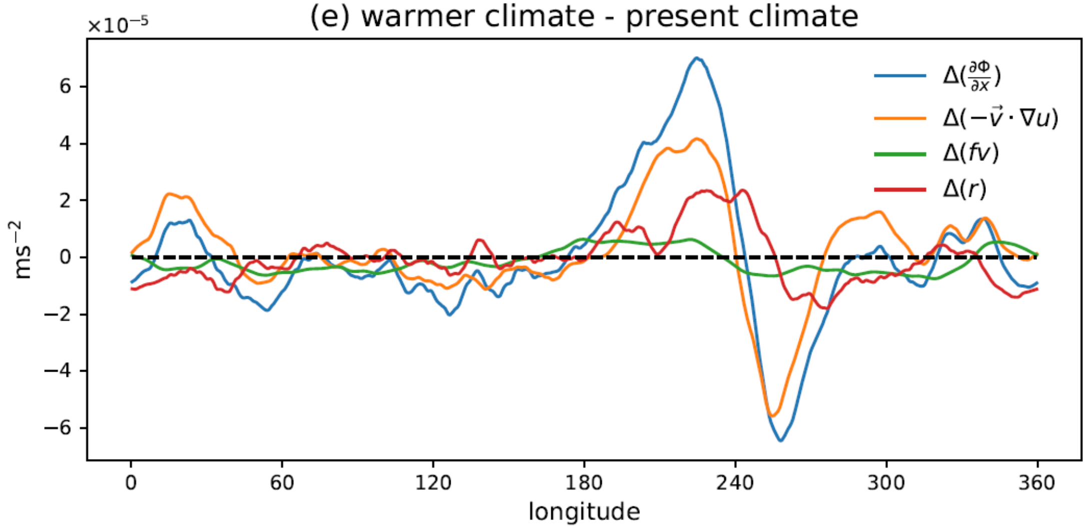

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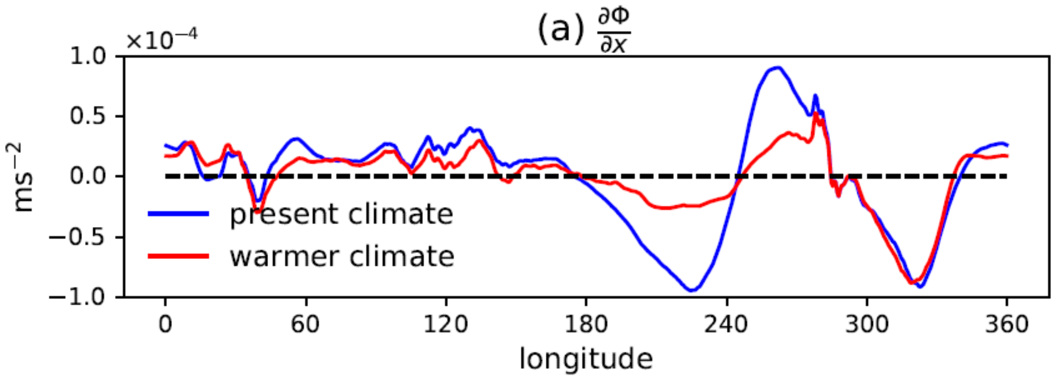
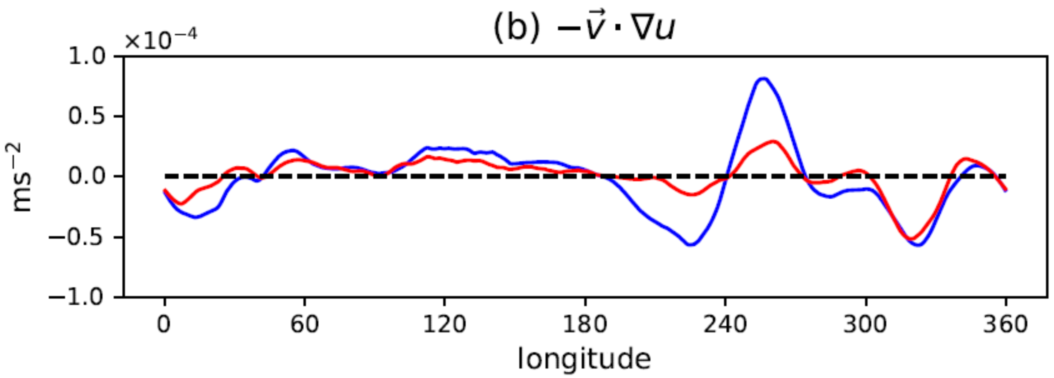
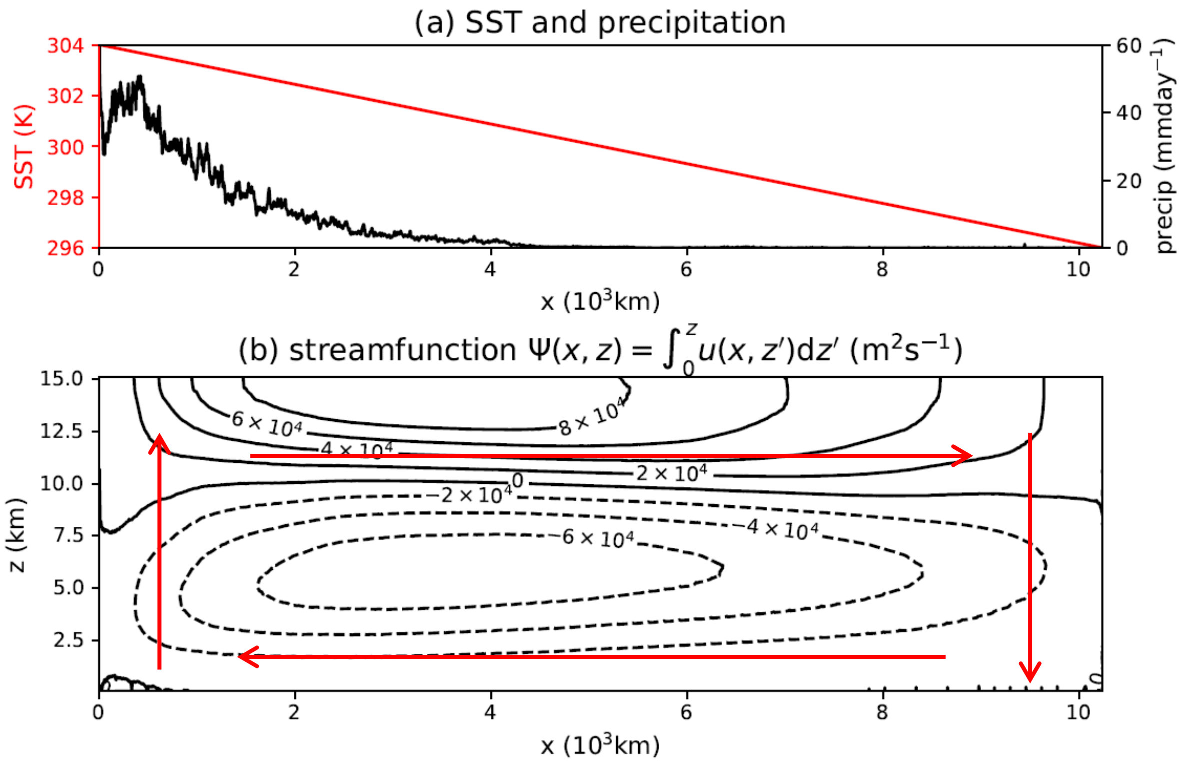




CRM verification: weaker circulation → weaker T gradient



Cloud Resolving model (CRM)
System for atmospheric modeling (SAM)

2D mock Walker circulation
Prescribed linear SST
Prescribed uniform radiative cooling rate
Solid wall boundary conditions

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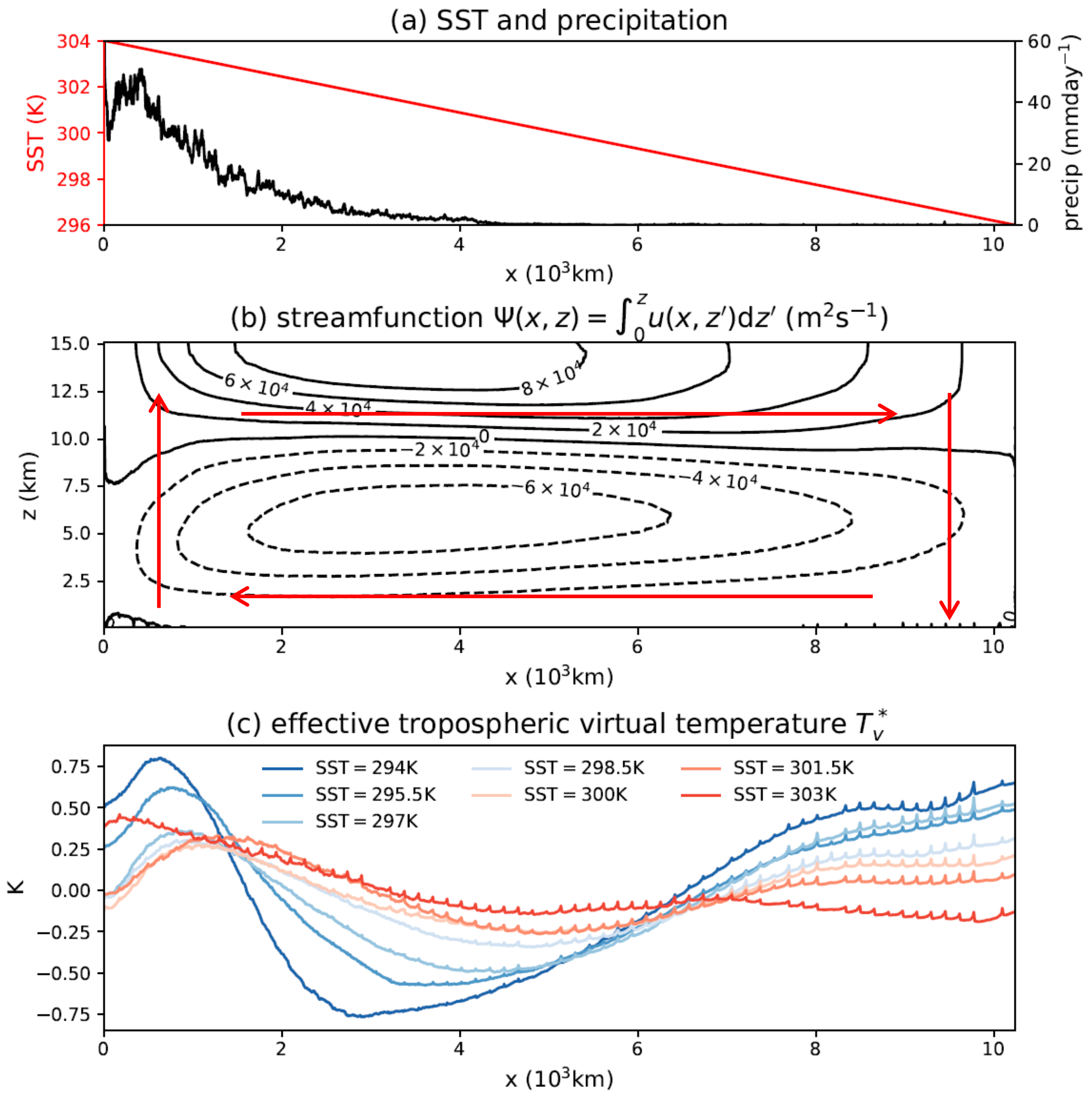

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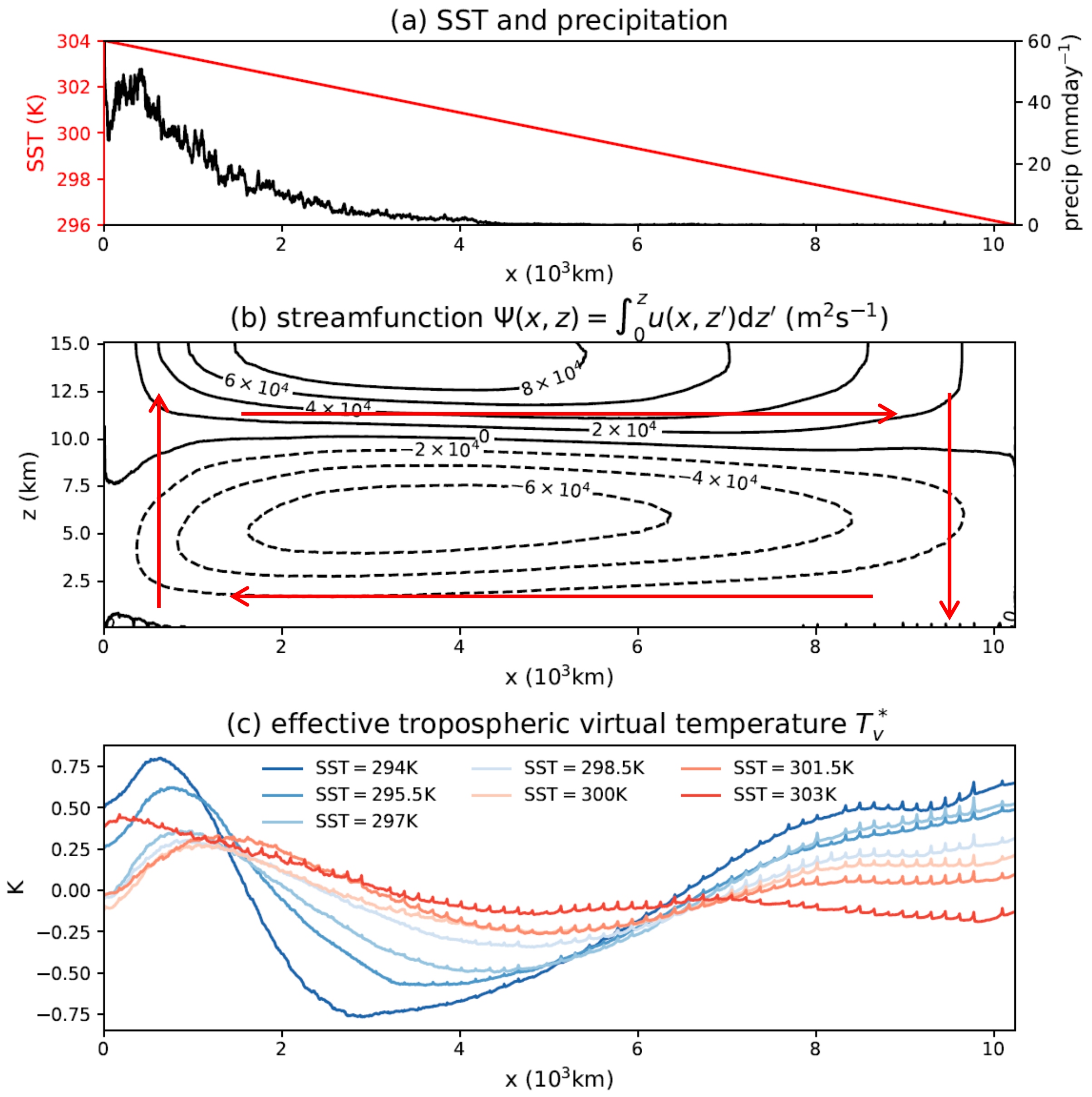

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1. warmer SST → **weaker circulation** → weaker T gradient?

2. quantitative scaling?

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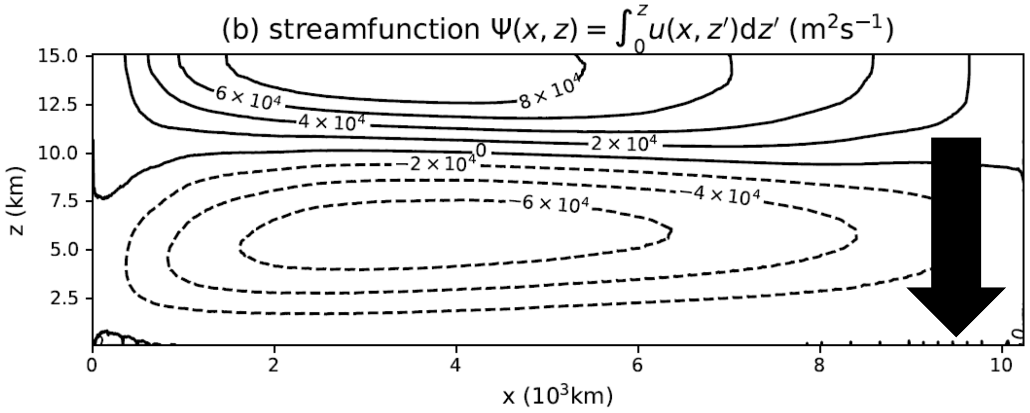
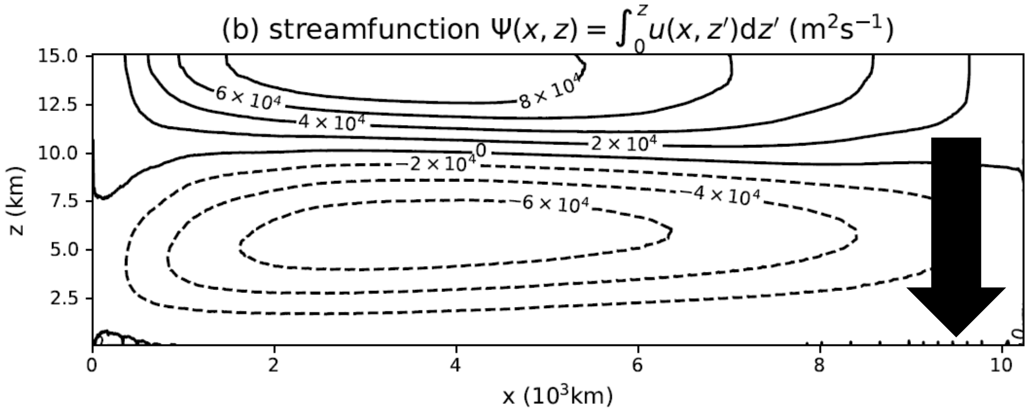


Cold end subsiding velocity: $w = Q_{rad}/S$

$Q_{rad}(< 0)$: radiative cooling rate

$S = \frac{\partial T}{\partial z} + \frac{g}{c_p}$: stability

CRM verification: weaker circulation → weaker T gradient



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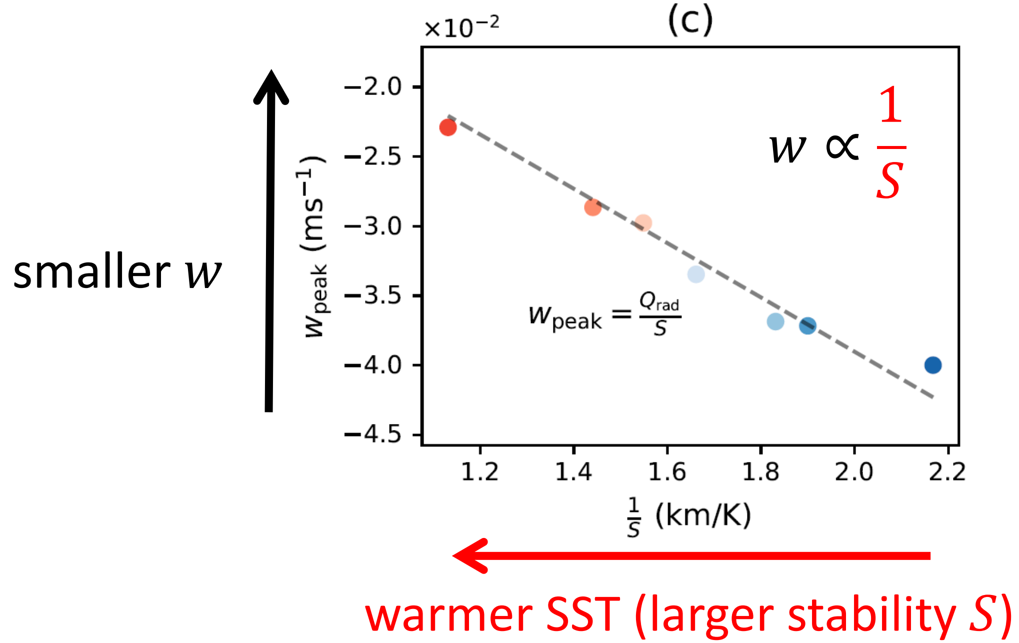

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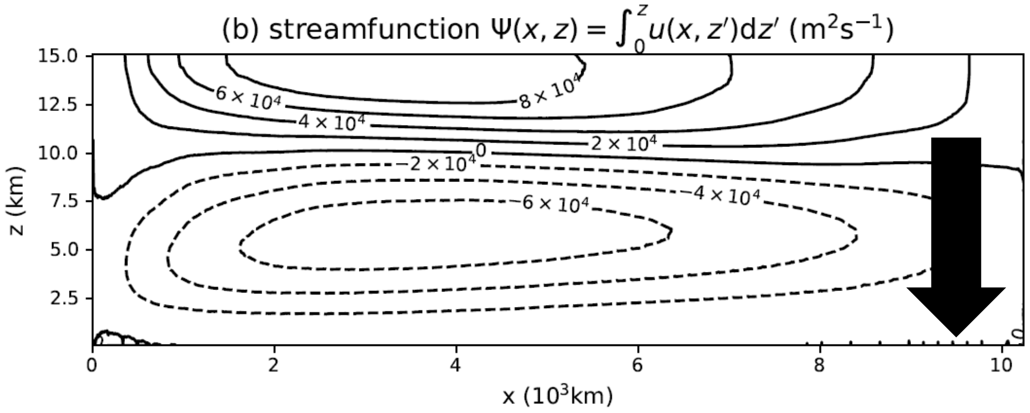

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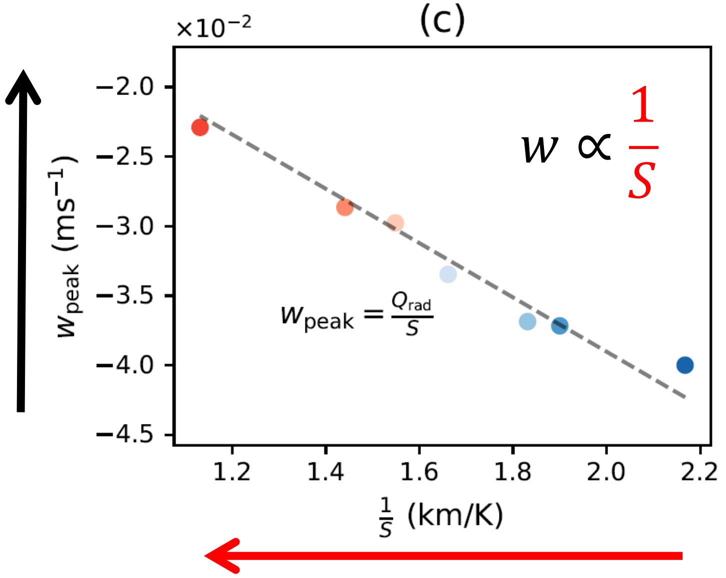
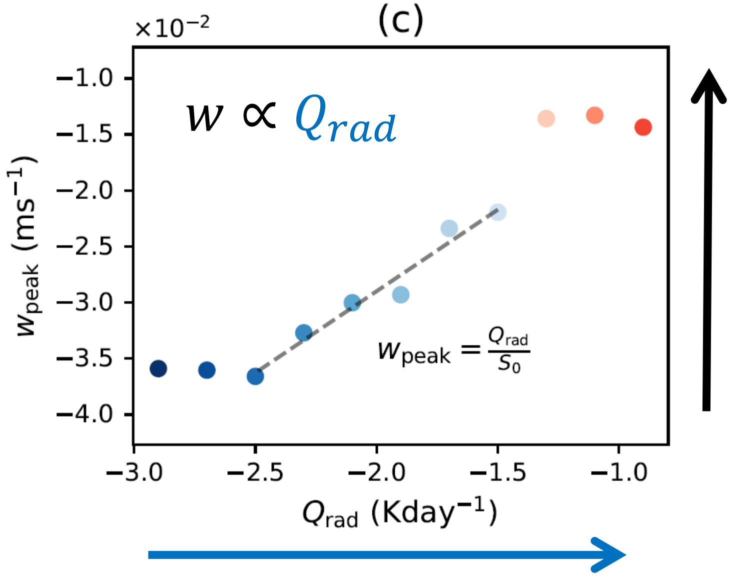
smaller w



warmer SST (larger stability S)

2. Fix SST (fix stability S)



smaller w

Smaller Q_{rad}

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$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} \approx u\frac{\partial u}{\partial x}$$

$$\frac{\mathrm{d}p}{\mathrm{d}z} = -\frac{p}{R_d T_v} g$$

(quantitative predictions)

CRM verification: weaker circulation → weaker T gradient

(quantitative predictions)

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \longrightarrow \frac{U}{L} \sim \frac{W}{H}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} \approx u\frac{\partial u}{\partial x} \longrightarrow \frac{\partial p}{\partial x} \propto U^2$$

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(quantitative predictions)

See our preprint for estimation of the coefficient

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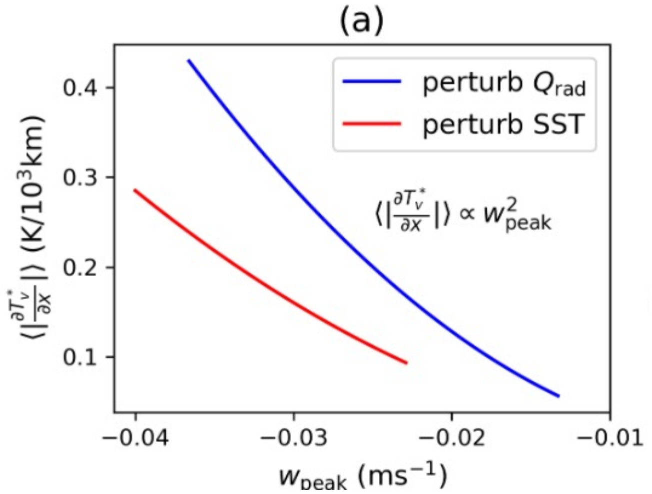

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(quantitative predictions)

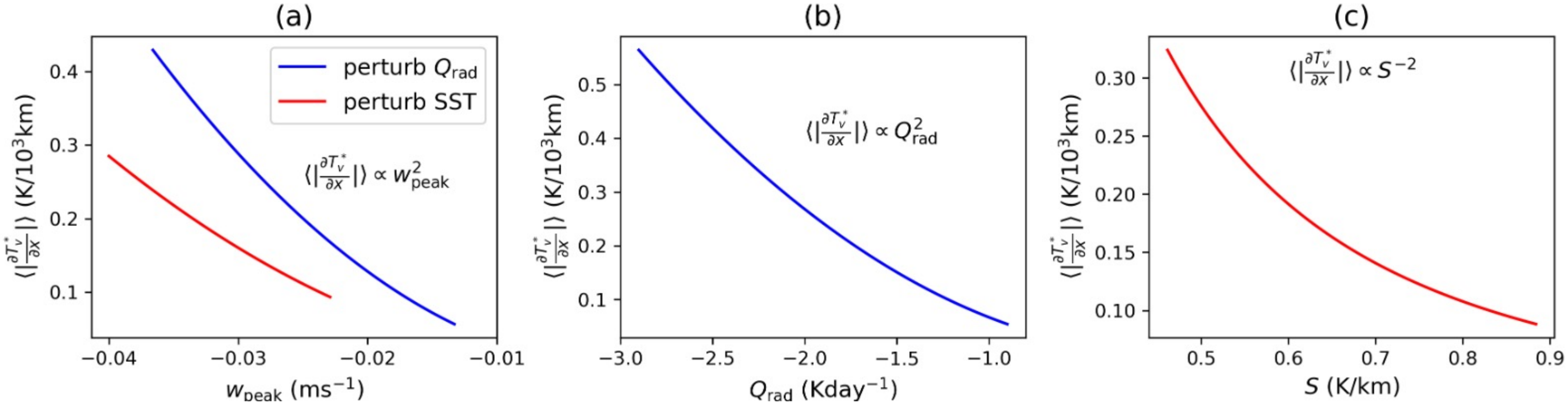
See our preprint for estimation of the coefficient

$$w = Q_{rad}/S$$

$\frac{\partial T_v}{\partial x} \propto w^2$

Fix S, $\frac{\partial T_v}{\partial x} \propto Q_{\mathrm{rad}}^2$

Fix Q_{rad}, $\frac{\partial T_v}{\partial x} \propto S^{-2}$



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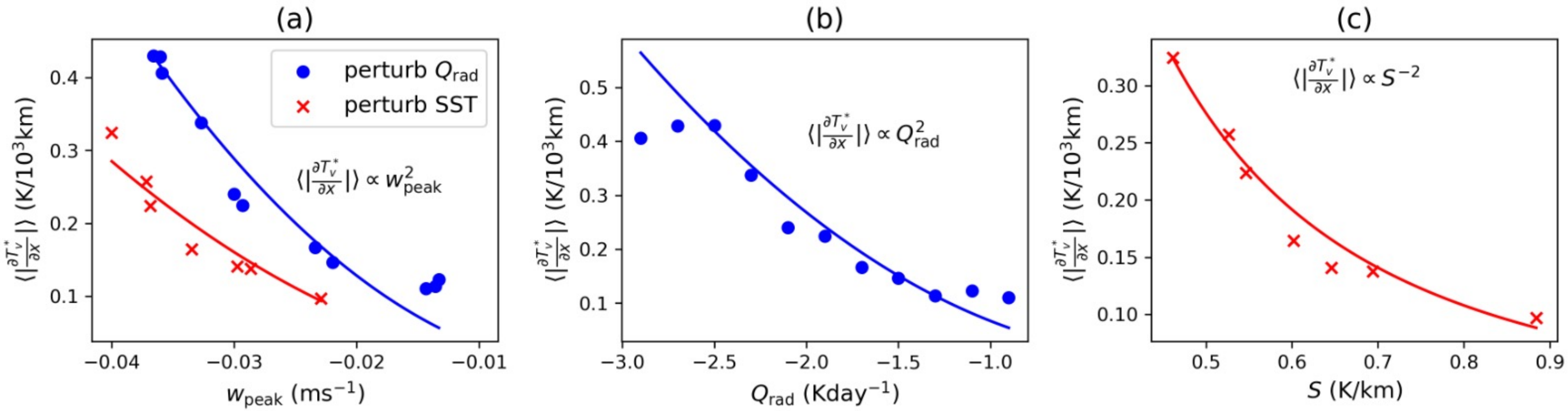
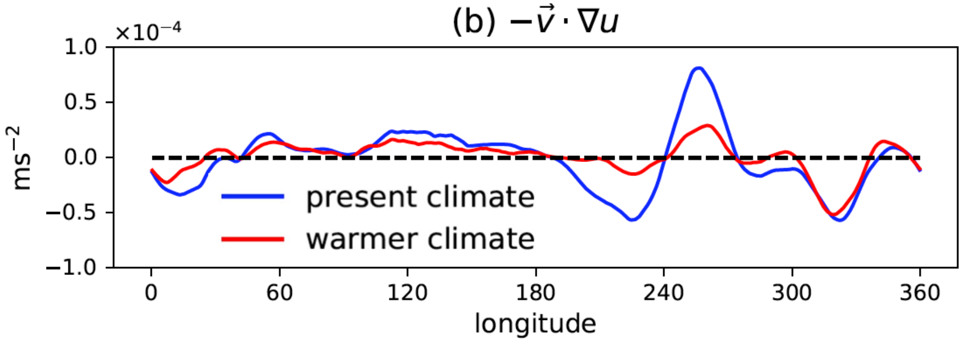


Consistent with model results! 😄

Conclusion: weaker circulation causes weaker tropical free troposphere temperature gradient in a warmer climate



Weaker circulation $w = \frac{Q_{rad}}{S}$

(real-world: S increases faster than Q_{rad})

Weaker momentum advection

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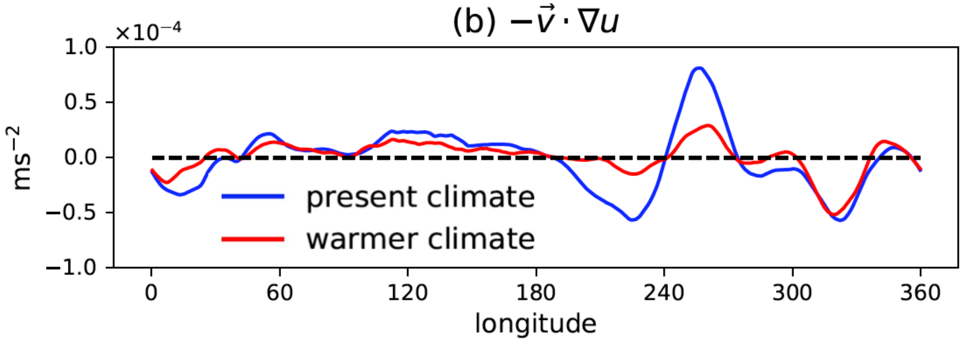

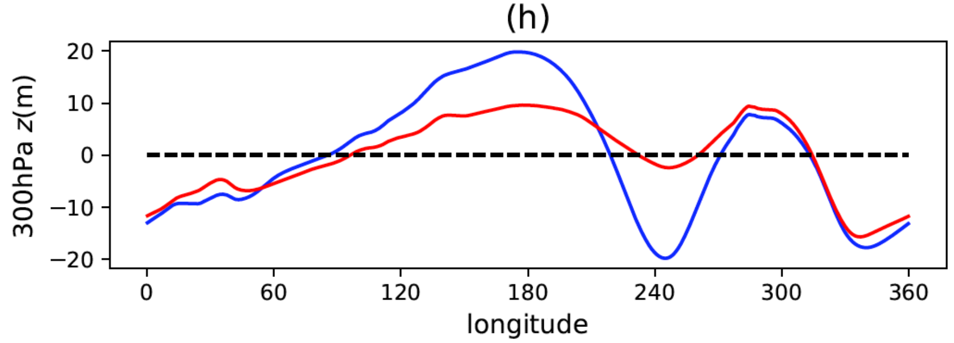

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Weaker momentum advection

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$$\Delta(-\frac{\partial \Phi}{\partial x}) + \Delta(-\vec{v}\cdot\nabla u) \approx 0$$

Weaker pressure gradient

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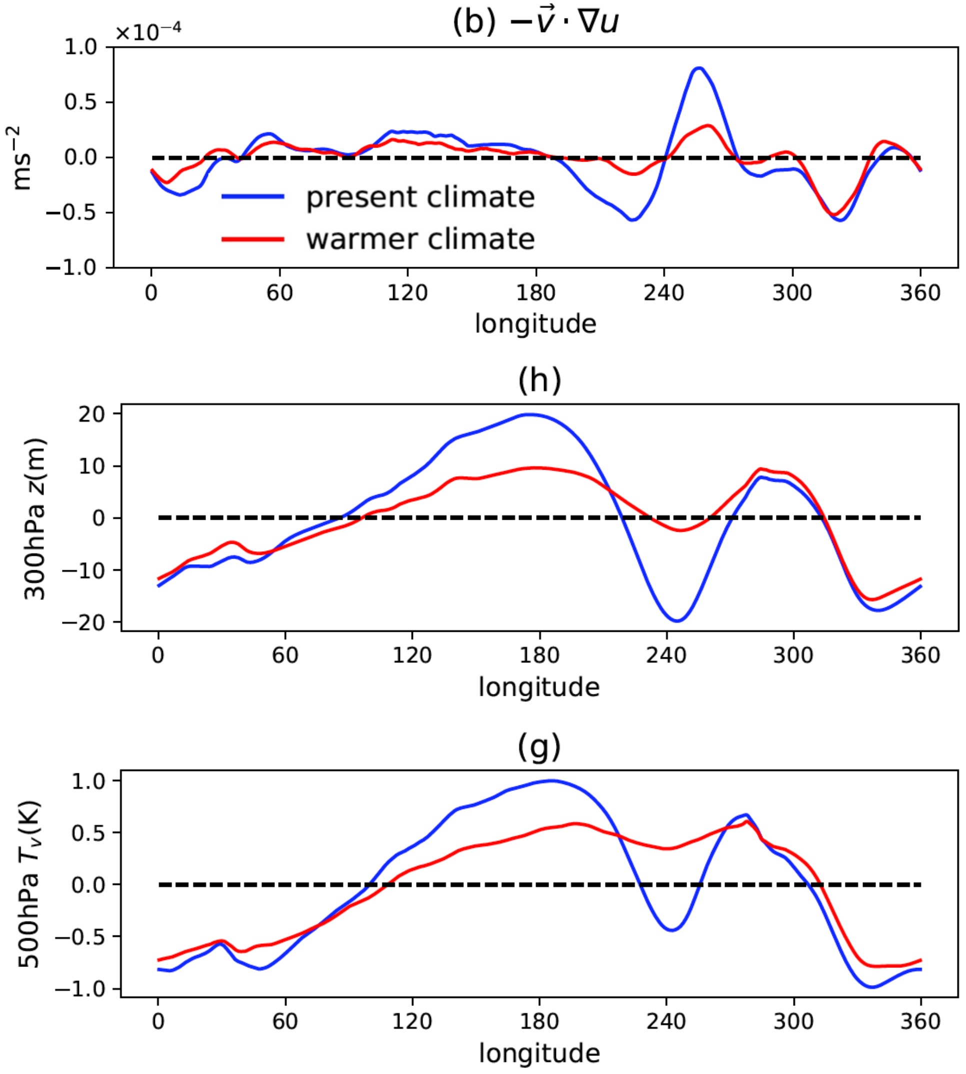

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Momentum equation

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Weaker pressure gradient

Hydrostatic equation

Weaker temperature gradient

The WTG approximation will be more accurate in the future! 😄

Contact me:
hengquan@princeton.edu
https://heng-quan.github.io

The weak temperature gradient approximation (Charney 1963, *JAS*)

$$\frac{\partial \vec{u}}{\partial t} + \vec{v} \cdot \nabla \vec{u} + f\hat{z} \times \vec{u} = -\frac{1}{\rho}\nabla_z p$$, large-scale $L \approx 1000km$

midlatitude free-troposphere

$$f\hat{z} \times \vec{u} \approx -\frac{1}{\rho}\nabla_z p$$

$$fU \sim -\frac{\delta p}{\rho L}$$

$$\frac{\delta p}{p} \sim \frac{\rho f U L}{p} \sim \frac{fUL}{gH} \approx \frac{10^{-4} \times 10 \times 10^6}{10 \times 10^4} = 10^{-2}$$

$$\mathrm{d}p = -\frac{p}{RT} g \mathrm{d}z \rightarrow \frac{\delta T}{T} \sim \frac{\delta p}{p} \approx 10^{-2}$$

If $T \approx 250K$, horizontal variation
$\delta T \approx \mathbf{2.5K}$ over $L \approx 1000km$

Tropical free-troposphere

$$\vec{v} \cdot \nabla \vec{u} \approx -\frac{1}{\rho}\nabla_z p$$

$$\frac{U^2}{L} \sim -\frac{\delta p}{\rho L}$$

$$\frac{\delta p}{p} \sim \frac{\rho U^2}{p} \sim \frac{U^2}{gH} \approx \frac{10^2}{10 \times 10^4} = 10^{-3}$$

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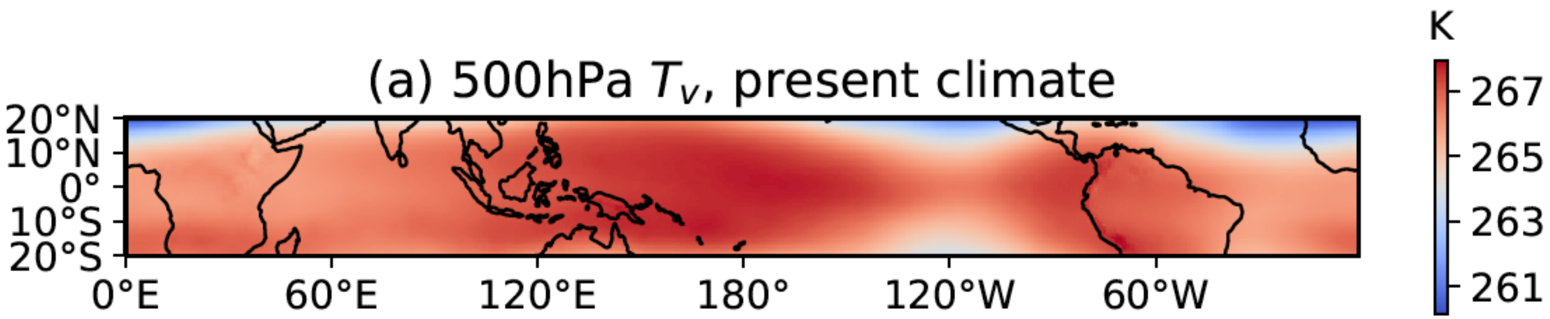
$\delta T \approx \mathbf{0.25K}$ over $L \approx 1000km$

The weak temperature gradient approximation (Charney 1963, *JAS*)

- Tropical free troposphere cannot maintain large horizontal temperature gradient due to the smallness of the Coriolis parameter close to equator (Charney 1963, *JAS*).

- Weak temperature gradient is NOT zero temperature gradient; Convective regions can be ~3K warmer than subsiding regions (Bao et al., 2022, *JCLI*).

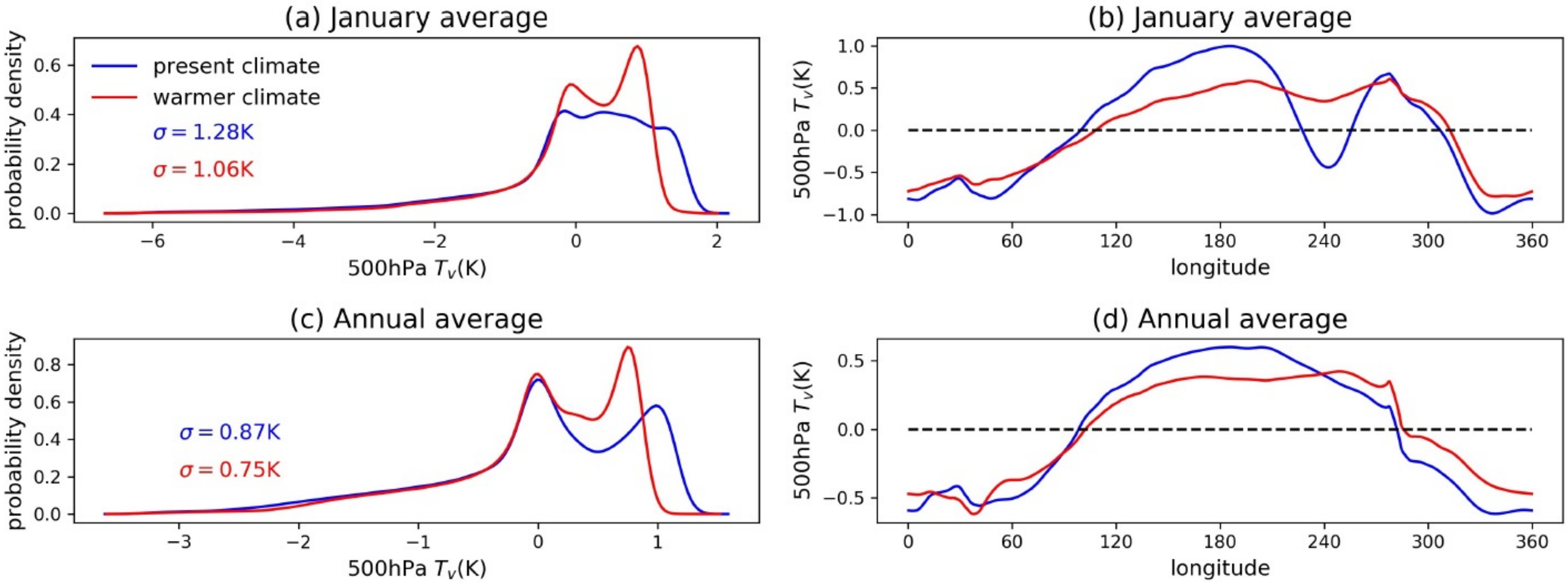


$\delta T \approx 0.25K$ over $L \approx 1000km$
$\delta T \approx 2.5K$ over $L \approx 100°$

Virtual temperature $T_v = T(1 + 0.61q)$ where q is specific humidity

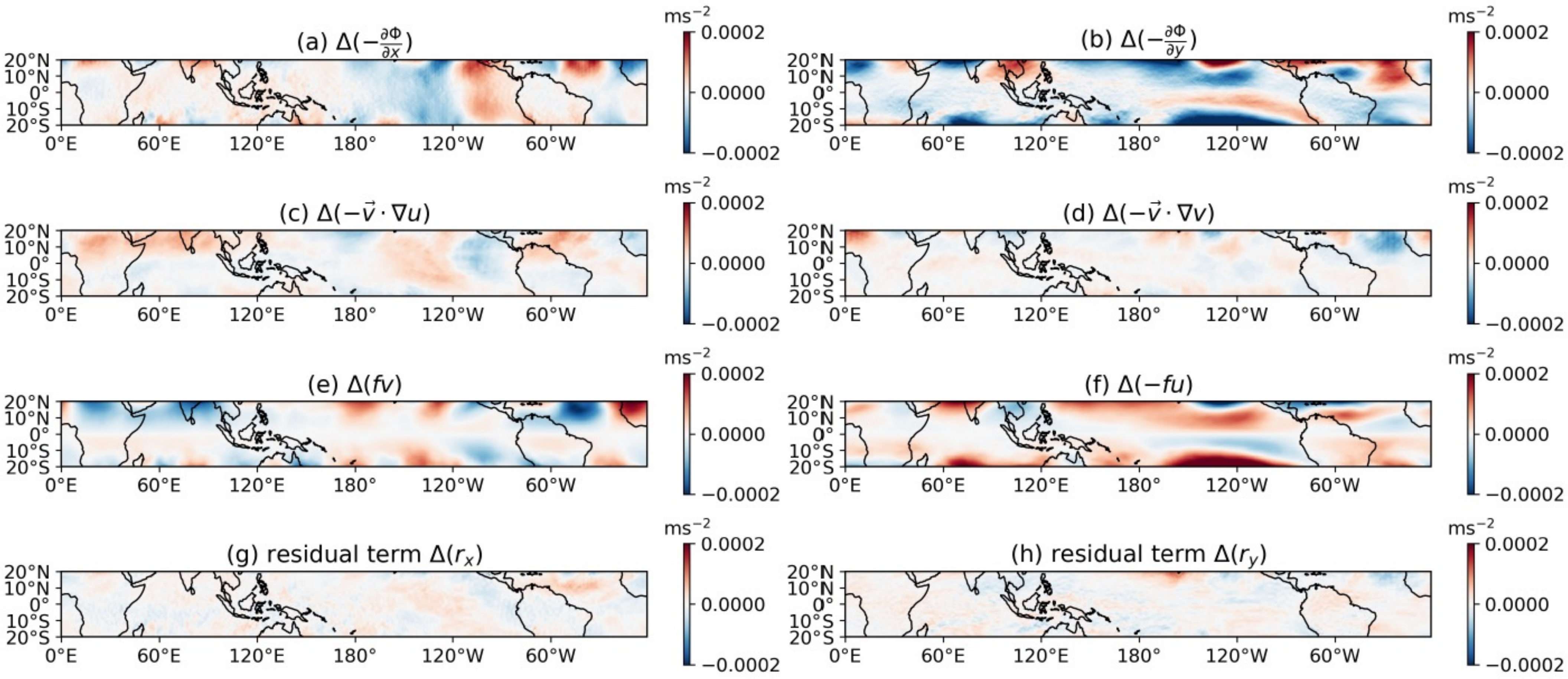
Question: How will tropical free troposphere temperature gradient respond to global warming?

GCM shows weaker temperature gradient in a warmer climate



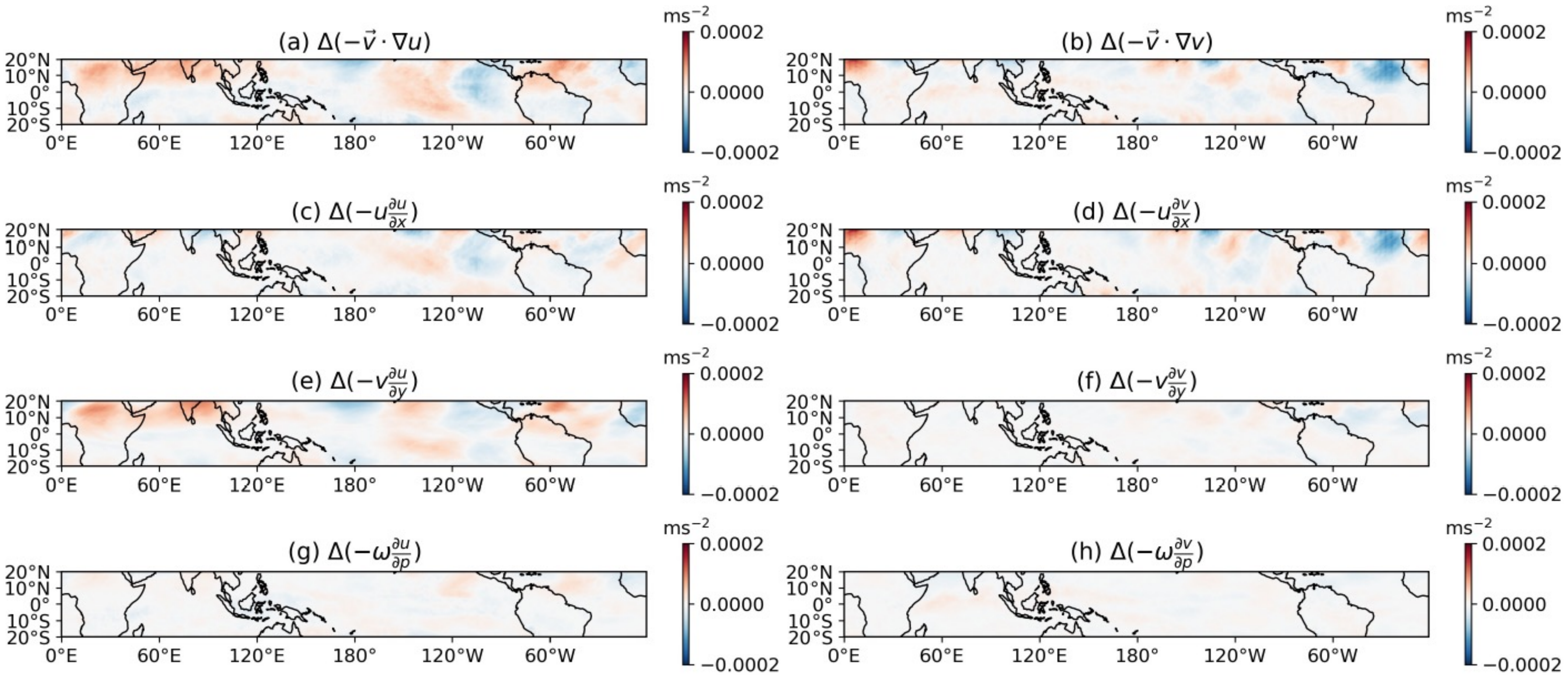
Annual mean results similar to January mean results

Hypothesis 3: circulation weakening ✅



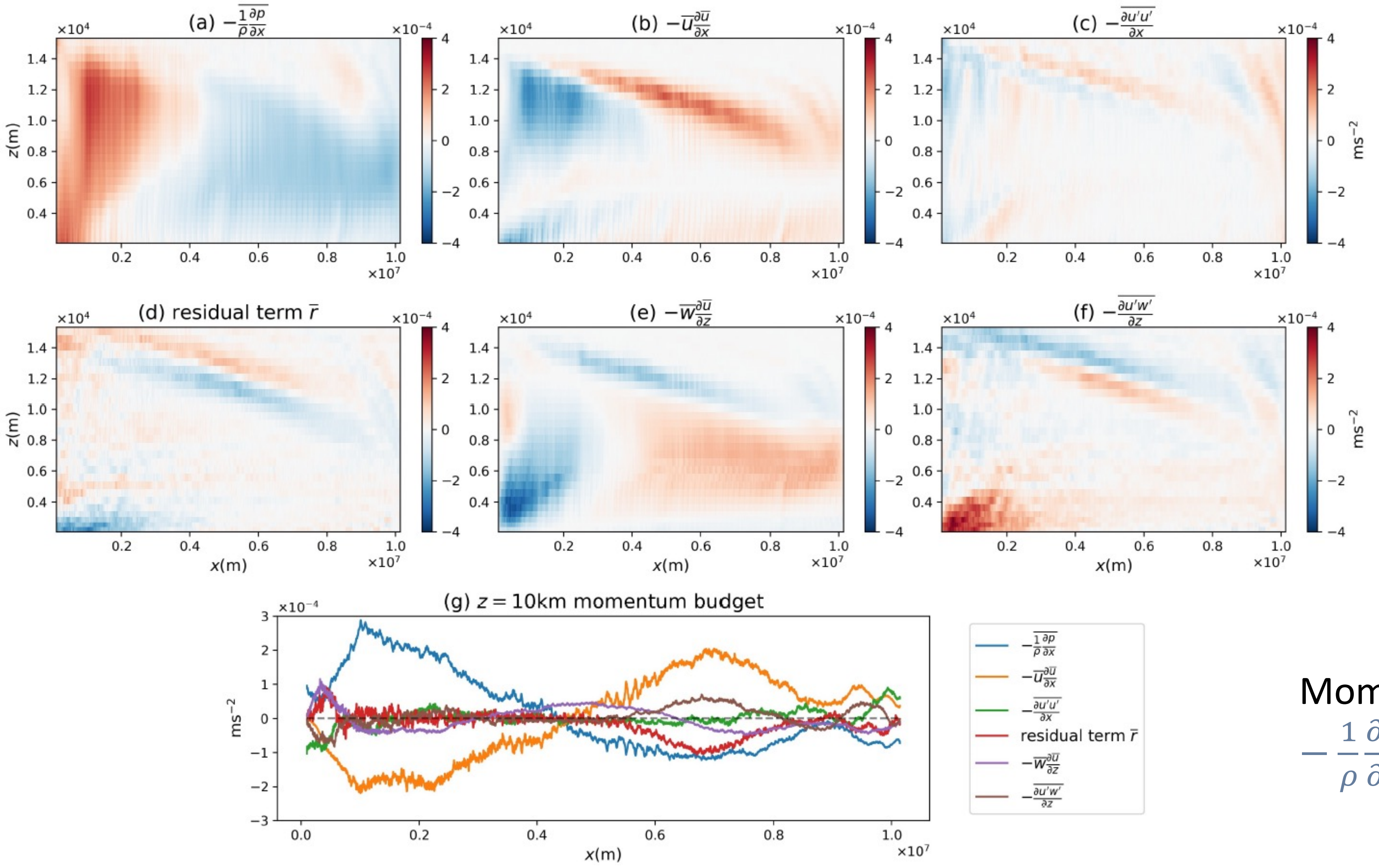
Weaker meridional pressure gradients balanced by weaker Coriolis force (i.e. weaker westerly)

Hypothesis 3: circulation weakening ✅



Decomposition of momentum advection terms

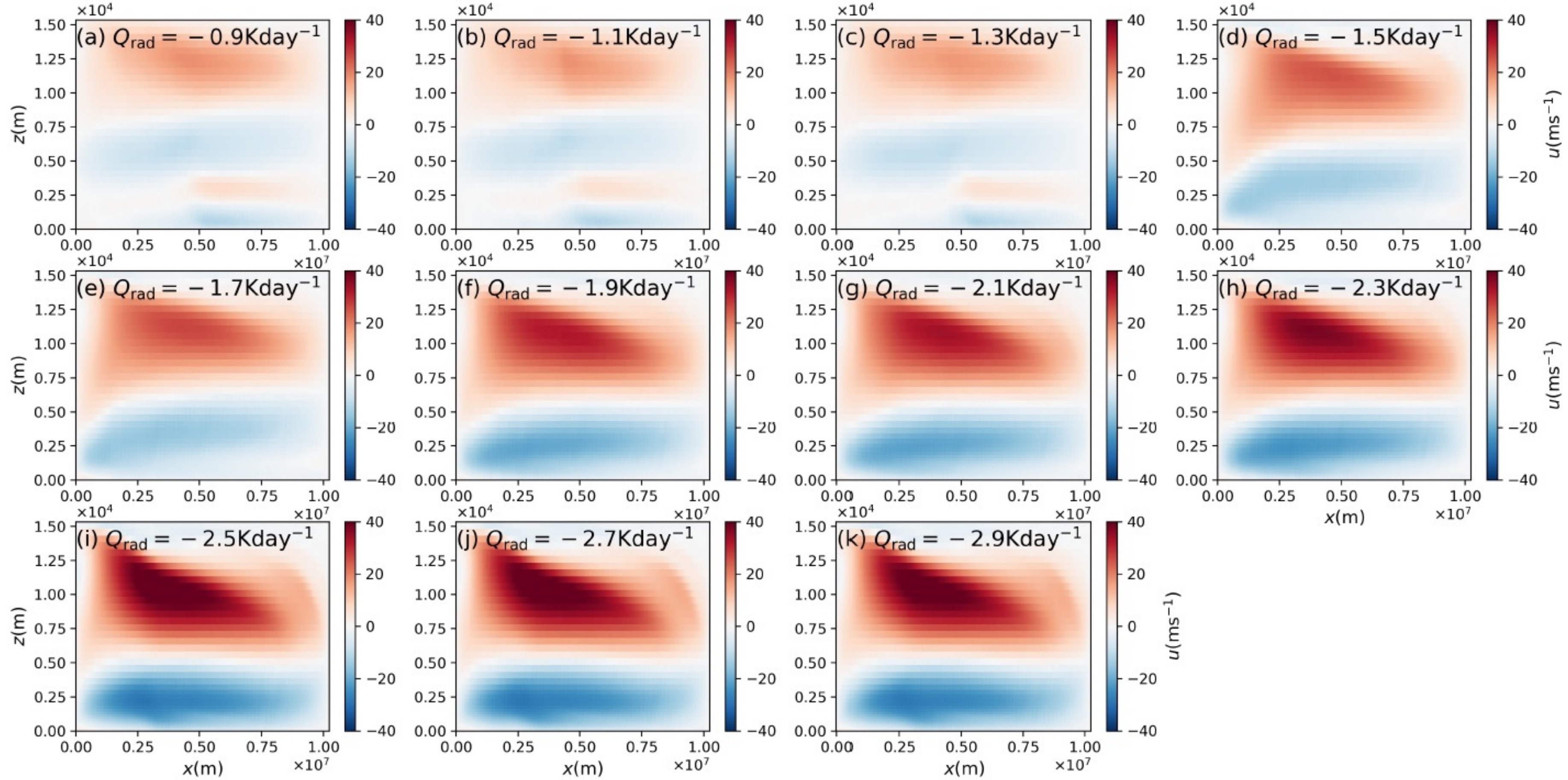
CRM verification: weaker circulation → weaker T gradient



Momentum equation:

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} + \left(-\bar{u}\frac{\partial \bar{u}}{\partial x}\right) \approx 0$$

CRM verification: weaker circulation → weaker T gradient



2-D horizontal velocity structures show **regime shift** when Q_{rad} becomes very small